CHAPTER 2

Proportional Reasoning and Variation

Many real-life problems have either no answer or more than one valid answer. In *Discovering Algebra,* students see some exercises with more than one answer. For example, in Lesson 2.1, Exercise 7 instructs, "Write three other true proportions using these four integers." Continue to focus on having your student explain his or her reasoning rather than on getting the "right answer." If the reasoning is good, then good answers will follow.

In Chapter 2, students get more comfortable with solving problems that they haven't been told how to solve. For example, in the Just Undo It! Investigation of Lesson 2.8, students are asked to explain a number trick rather than being given an explanation. This approach builds skills and confidence in solving problems.

Content Summary

Chapter 2 continues the modeling journey. Students focus on writing equations and solving equations for an unknown number. Chapter 2 equations are *proportions,* which involve *ratios.* Students learn about direct and indirect variation and learn to solve equations by *undoing* operations.

Ratios

A *ratio* is a comparison of one quantity relative to another by division, so *Discovering Algebra* shows most ratios as fractions. Students might already think of fractions as parts of a whole (2 of 3 equal parts), as implied division (2 divided by 3), or as numbers (a value between 0.5 and 1). Thinking about ratios can be challenging. In fact, when working mathematicians disagree about something, such as a question of probability, a ratio is often involved.

Proportions

A *proportion* is an equation stating that two ratios are equal. Four quantities are involved in a proportion, two in each ratio. When one of these quantities is unknown, you can solve the proportion to find it. In their book, students see how proportions can represent various situations, including conversion of measurement units and capture-recapture studies.

If you solved proportions in school, you might have used a method called "cross-multiplying." It's an efficient method when used correctly, but students tend to latch on to it without understanding why it works, and then often use it in situations where it doesn't apply. To build understanding and reinforce thinking, *Discovering Algebra* avoids cross-multiplication. You can solve all proportion equations at this level by multiplying to *undo* the division, perhaps after inverting the proportion. See the Multiply and Conquer Investigation on pages 97 and 98.

Dimensional Analysis

Making conversions between units in different systems of measurement is an important skill for school and for daily life. For example, you might need to compare metric units with English units, fluid measurements with cup measurements, or miles per hour with feet per second. Dimensional analysis is a strategy that leaves no doubt about whether you should multiply or divide by a conversion factor. See Example B on pages 109 and 110.

(continued)

Chapter 2 • Proportional Reasoning and Variation (continued)

Direct Variation

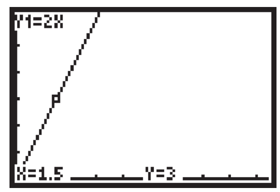
Studying direct variation is an excellent way to deepen students' understanding of linear relationships and to prepare them for further study of linear equations in Chapters 3 through 5.

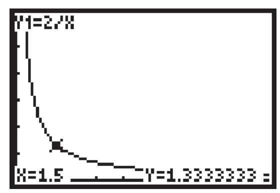
A *direct variation* is an equation such as $y = \frac{3}{2}x$ or $y = -5x$. These equations are equivalent to the proportions $\frac{y}{x} = \frac{3}{2}$ and $\frac{y}{x} = \frac{-5}{1}$. In general, a direct variation is an equation of the form $y = kx$, where k is a number.

If the constant (unchanging) number k is considered as a ratio with denominator 1, this number is called a *rate*. Many conversion factors, such as 5280 $\frac{feet}{mile}$, are rates. So are speeds, in $\frac{miles}{hour}$ or $\frac{feet}{second}$. When speed (rate) is constant, the equation relating distance and time traveled is a direct variation. A graph of (*time*, *distance*) of an object moving at a constant speed is a line.

Inverse Variation

Chapter 2 also considers inverse variation, although an inverse variation is not a linear equation. Rather, an *inverse variation* is an equation of the form $y = \frac{k}{x}$, where k is a constant number. These equations and their graphs will reappear in Chapter 8.





Graphs of Variations

Variations are equations, so they have graphs. Students see the graphs of both direct and indirect variations in this chapter, although only the graphs of direct variations are straight lines. Students learn that k, the *constant of variation*, is related to the steepness of the line. To keep students focused on the rate-of-change concept, the explanation for the mathematical term *slope* is delayed until Chapter 4.

Summary Problem

You and your student might discuss the following problem. It's a good problem to revisit several times while working through the chapter.

What problem situations can you represent by the proportion $\frac{x}{150} = \frac{17}{20}$, or by an inversion of one or both sides of the proportion?

Questions you might ask in your role as student to your student include:

- Could this proportion represent a capture-recapture problem?
- Can you change the proportion to represent a percent problem?
- Can you change the proportion to represent a problem about measurement?
- Can you present several methods for solving the problem?

Sample Answer

Many different answers are possible. Encourage students to be creative and to discuss the interpretation of each number in the proportion by using different contexts. The proportion might represent a capture-recapture problem. In a sample of 20 fish, 17 were found to be tagged. If a scientist predicted that 150 fish total are in the lake, how many fish did the scientist tag? To represent a percent problem, the ratio $\frac{17}{20}$ might be interpreted as a quiz score, whereas $\frac{x}{150}$ is the equivalent score on the final exam. The ratio $\frac{85}{100}$, or 85%, is equivalent to both scores. For measurement, the scale on a map might show that 20 cm represents 150 km. If two cities are 17 cm apart on the map, how many km apart are they?

Remind students to multiply by 150 to undo the division by 150.

Chapter 2 • Review Exercises

Name ____ **Period** ____________ **Date** ________________

1. *(Lessons 2.1, 2.2)* Write a proportion to answer each question. Then solve the proportion.

a. What number is 30% of 75?

b. What percent of 16 is 125?

c. 13 is 0.5% of what number?

2. *(Lesson 2.3)* One kilogram (kg) is approximately 2.2 pounds (lb). Which is heavier—a 17 lb object or an 8 kg object?

3. *(Lessons 2.4, 2.5)* Determine whether each relationship describes direct or inverse variation. In each case, write an equation in $y=$ form, and then graph it on your calculator.

a. $xy = 0.5$

b. $y = 0.5x$

c. The relationship between the number of pages of a book Ari reads and the amount of time it takes him to read it, if he reads at a constant rate of 2 minutes per page.

d. The relationship between the speed a car travels and the amount of time it takes to cover a distance of 30 mi.

4. *(Lessons 2.7, 2.8)* Identify the order of operations. Then create an undo table and use it to solve the equation.

$$\frac{2x - 1}{3} + 5 = 8$$

5. *(Lesson 2.8)* A car rental company charges a flat rate of \$30, plus \$0.20 for every mile driven.

a. Write an equation that relates the cost of renting a car to the number of miles driven.

b. Gloria rented a car from this company for a weekend trip, and her bill came to \$58.40. How many miles did Gloria drive?

SOLUTIONS TO CHAPTER 2 REVIEW EXERCISES

1. a. $\frac{x}{75} = \frac{30}{100}$ The proportion.

$x = \frac{30}{100} \cdot 75$ Multiply by 75 to undo the division.

$x = 22.5$ Multiply and divide.

22.5 is 30% of 75.

b. $\frac{x}{100} = \frac{125}{16}$ The proportion.

$x = \frac{125}{16} \cdot 100$ Multiply by 100 to undo the division.

$x = 781.25$ Multiply and divide.

125 is 781.25% of 16.

c. $\frac{13}{x} = \frac{0.5}{100}$ The proportion.

$\frac{x}{13} = \frac{100}{0.5}$ Invert the proportion.

$x = \frac{100}{0.5} \cdot 13$ Multiply by 13 to undo the division.

$x = 2600$ Multiply and divide.

13 is 0.5% of 2600.

2. Use proportions to convert both weights to the same units. For example, convert 8 kilograms to pounds. Let x be the weight of the 8 kg object, in lb.

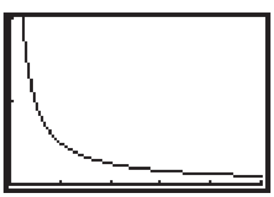
$\frac{2.2 \text{ lb}}{1 \text{ kg}} = \frac{x \text{ lb}}{8 \text{ kg}}$ The proportion.

$(8 \text{ kg}) \cdot \frac{2.2 \text{ lb}}{1 \text{ kg}} = x \text{ lb}$ Multiply by 8 to undo the division.

$x = 17.6$ Multiply and divide.

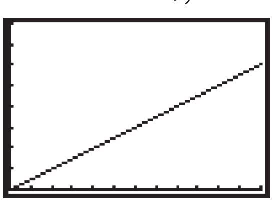
The 8 kg object is heavier.

3. a. Inverse variation; $y = \frac{0.5}{x}$.



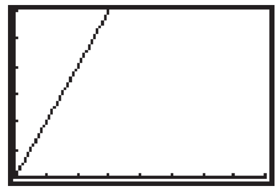
[0, 5, 1, 0, 2, 1]

b. Direct variation; $y = 0.5x$.



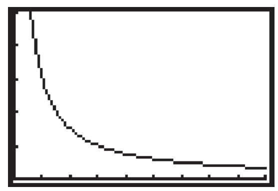
[0, 12, 1, 0, 8, 1]

c. Direct variation; $y = 2x$, where x is the number of pages Ari reads, and y is the number of minutes he takes to read them.



[0, 40, 5, 0, 30, 5]

d. Inverse variation; $y = \frac{30}{x}$, where x is the speed of the car in mi/h, and y is the number of hours it takes the car to cover 30 mi.



[0, 90, 10, 0, 5, 1]

4. $x = 5$

Equation: $\frac{2x-1}{3} + 5 = 8$		
Description	**Undo**	**Result**
Pick x.		5
· (2)	/ (2)	10
− (1)	+ (1)	9
/ (3)	· (3)	3
+ (5)	− (5)	8

5. a. $y = 30 + 0.2x$, where x is the number of miles driven and y is the cost of the car rental.

b. 142 mi

Equation: $30 + 0.2x = 58.40$		
Description	**Undo**	**Result**
Pick x.		142
· (0.2)	/ (0.2)	28.40
+ (30)	− (30)	58.40